AN EVALUATION OF DIGITAL-FILTER BASED SYNTHETIC TURBULENCE GENERATION METHODS AND IMPROVEMENTS TO THEIR QUANTIFIED DEFICIENCIES

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As emphasized by [4], the governing equations for turbulent flows are extremely sensitive to inlet boundary conditions. Lorenz [4] showed, for example, that an alteration in the streamwise velocity component initial condition merely at a level of $\mathcal{O}\left(10^{-6}\right)$ yields diverse instantaneous realizations in turbulent convection mechanisms. Large eddy simulation (LES) is also subject to such sensitivity because of the direct computation of the governing equations for time-dependent large-scale turbulent motions. Inlet boundary conditions for LES, therefore, must be carefully modelled. Yet, theoretical and practical inlet turbulence generation is proved to be difficult mainly due to the complex nature of turbulence; hence, resulting in various methods.

One important category of such methods is the *digital-filter based synthetic turbulence generation methods*, originally proposed by [3], (hereafter, DFMs) refer to which accept a set of *target* statistics and a discrete random signal as input, and transform them mostly through mathematical techniques into a new deterministically and statistically different signal representing the fluctuating component of a Reynolds decomposed turbulence parameter, $\phi'(\mathbf{x}, t)$. The transformation is performed by a train of arithmetic operations, which as a whole is often called a *discrete filter operator* (*filter*). Attributes and sequence of operations are arranged by the *target* statistics, so that the *realized* statistics of the new signal may match the *target*. The major advantage of DFMs in comparison to the other methods is the easiness of their code implementations and their relatively low computational cost for a similar level of fidelity for the generated turbulence realizations.

Despite the use of DFMs across a broad range of LES applications in the literature, the relevant literature arguably lacks systematic and complete conclusions/recommendations regarding inner parameters/mechanisms of DFMs. For instance, quantitative examinations for their modelling assumptions, input-output relations, best numerical implementation/usage practices, and extensive comparative analyses across their variants are, in general, either unavailable or unorganized. Lack of knowledge on such issues may, however, hamper theoretical and practical improvements for DFMs, and their correct usage.

The aim of this research study is, thus, to systematically explore patterns in parameters/mechanisms of DFMs to fill these knowledge gaps, and to propose and evaluate possible improvements. For this purpose, the objective is set to investigate each building-block assumption of DFMs in a consecutive order, examine outcome realizations to reveal capabilities and deficiencies of the method, and search for new extensions/inversions to remedy the quantified deficiencies.

Three methods representing general capabilities of DFMs are tested with and without LES: $i.$ [3], $ii.$ [2]'s forward stepwise method (FSM), and $iii.$ [5]'s hybrid FSM-DFM. Their common point is that their applicability to most LES cases is possible unlike, for example, [1] is limited to homogeneous shear flows in practice. Additionally, the test beds of the methods involve: $i.$ homogeneous isotropic turbulence, $ii.$ homogeneous shear flow, $iii.$ channel flow, and $iv.$ backward facing step flow, each of which helps to focus one isolated aspect of turbulent flows.

Initial quantifications showed two principal deficiencies in outcomes of DFMs: $i.$ they are limited to Gaussian distributions, and $ii.$ they lose energy near cut-off scales. In addition, initial work suggested that DFMs may provide more sophistication at a lower computational cost. For the first time, accordingly, a new method was proposed in order to improve DFMs to produce non-parametric non-Gaussian turbulence processes. Furthermore, two new methods for DFMs' inner computations were developed, one reduced the floating-point operations per time-step (FLOPT) from $\mathcal{O}\{N^6\}$[1] to $\mathcal{O}\{N^3(3N)\}$, and the other reduced the FLOPT from $\mathcal{O}\{N^6\}$ to $\mathcal{O}\{N^3 log_3(N)\}$ for a typical LES computation.

In the final work, the characteristics/quantifications of DFMs' parameters/assumptions/mechanisms, arguably in the largest scale in the literature to date, and, the methods-yielding-promising-results, which will undergo formalization and more extensive tests, will be presented.

References

[1] M. Fathali, J. Meyers, G. Rubio, S. Smirnov, and M. Baelmans. Sensitivity analysis of initial condition parameters on the transitional temporal turbulent mixing layer. *Journal of Turbulence*, **9**:N12, 2008.

[2] Y. Kim, Z. T. Xie, and I. P. Castro. A forward stepwise method of inflow generation for les. *Proceedings of the Sixth International Conference on Fluid Mechanics*, pages 134–136, 2011.

[3] M. Klein, A. Sadiki, and J. Janicka. A digital filter based generation of inflow data for spatially developing direct numerical or large eddy simulations. *Journal of Computational Physics*, **186**:652–665, 2003.

[4] S. B. Pope. *Turbulent Flows*. Cambridge University Press, 2000.

[5] Z. Xie and I. P. Castro. Efficient generation of inflow conditions for large eddy simulation of street-scale flows. *Flow, Turbulence and Combustion*, **81**:449–470, 2008.

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[1] N is the number of grid points along a coordinate axis.